STEPPER MOTOR POSITION CONTROL WITH OPTIMAL COMPENSATION

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Abstract

The paper presents a modeling and control methodology for designing high-performance optimal drive systems for stepper motor devices. The paper deals with a nonlinear model of an electric stepper motor and proposes a solution to the new intelligent/optimal position control problem with optimal energy compensation for the motor. The paper also proves that energy consumption and energy loss can be significantly reduced by employing optimal compensation while maintaining the same motor dynamics.

1 Introduction

Stepper motors are widely employed in various industrial applications requiring precise positioning, such as robotics, CNC machines, and medical devices [3]. Their ability to execute discrete steps with high accuracy makes them indispensable in modern automation systems. However, achieving optimal performance, particularly in terms of energy efficiency and reduced power losses still remains a challenge due to the nonlinear dynamics of the motor [1,2] and its control requirements. Traditional control strategies often overlook energy optimization, resulting in unnecessary energy losses and reduced system efficiency. The proposed methodology leverages nonlinear modeling of the motor to accurately capture its dynamics and employs intelligent compensation [4] to enhance efficiency without compromising performance. By managing both energy delivery and power loss minimization, this study provides a precise motion control with significantly improved energy efficiency [5].

2 Stepper motor model

As a control plant, a stepper motor model is chosen to apply and verify the proposed controller with an optimal compensator. The equations describing the electrical dynamics of the motor are as follows:

$$L\frac{di_1}{dt} = u_1 - Ri_1 - e_1, \tag{1}$$

$$L\frac{di_2}{dt} = u_2 - Ri_2 - e_2, \tag{2}$$

where

$$e_1 = -K_m\omega\sin(N\theta), \tag{3}$$

$$e_2 = -K_m\omega\cos(N\theta). \tag{4}$$

To complete the description, the motor mechanics are presented by angular motion equation:

$$J\frac{d\omega}{dt} + b\omega = T_e, \tag{5}$$

where

$$T_e = -K_m\left[i_1 - \frac{e_1}{R_m}\right]\sin(N\theta) + K_m\left[i_2 - \frac{e_2}{R_m}\right]\cos(N\theta). \tag{6}$$

where: e_1 and e_2 denotes the back electromotive forces (emfs), i_1 and i_2 are the phase winding currents, u_1 and u_2 are the phase winding voltages, K_m represents the motor torque constant, N is the number of teeth on each of the two rotor poles, R is the winding resistance, L is the winding inductance, R_m is the magnetizing resistance, b is the rotational damping, J is the inertia, ω is the rotor speed, θ is the rotor angle. The motor governing equations (1)-(6) can be described in state-space form. Moreover, employing state-dependent parametrization SDP [1,2], the motor model takes the parametrized form:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{A}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{7}$$

where state $\boldsymbol{x} = [i_1 \quad i_2 \quad \omega \quad \theta]^T$ and $\boldsymbol{u} = [u_1 \quad u_2]^T$.

3 Control system with SDRE compensation

The control system example presents a stepper motor drive using the hybrid two-phase model. The motor phases are fed by two H-bridge PWM (Pulse-Width Modulation) converters. The motor voltages are independently regulated by two hysteresis-based controllers that generate the drive signals from square-wave generator and optimal SDRE (State-Dependent Riccati Equation) compensator. The movement of the stepper drive is controlled by the STEP (Pulse Input) and DIR (Direction Control) signals received from position controller that generates signal sequence due to demanded position trajectory.

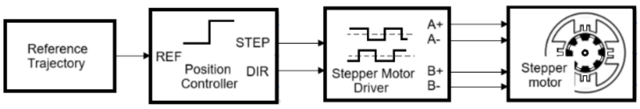

Fig 1: Stepper motor control system.

The motor driver contains controlled PWM voltage subsystem to generate a pulse-width modulated (PWM) voltage source. The block calculates the duty cycle based on the reference voltage. Then demanded duty cycle is:

$$\frac{V_{ref}-V_{min}}{V_{max}-V_{min}} \times 100\%, \tag{8}$$

where V_{ref} is reference voltage, V_{min} is the minimum reference voltage and V_{max} is the maximum reference voltage. The amplitude of PWM voltage is additionally controlled by optimal SDRE compensator, which increases or decreases actual amplitude of the voltage fed on the stepper motor. Shortly, the driver performs two important functions:

- generates PWM voltage control signal,
- compensate it optimizing a performance function.

The method of optimal compensation is well described in [1,4] and consists of finding optimal control that minimizes following performance function

$$J(\boldsymbol{u}) = \frac{1}{2}\int_0^\infty (\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{u}^T\boldsymbol{P}\boldsymbol{u})dt, \tag{9}$$

subject to nonlinear dynamics for affine system in the state-dependent coefficient (SDC) form (7), where $\boldsymbol{Q}$ is the symmetric, positive semi-definite weighting matrix for states, $\boldsymbol{P}$ is the symmetric, positive definite weighting matrix for control inputs.

In a process of solving SDRE problem, using the Hamilton Jacobi-Bellman theory and the necessary optimality condition, the following optimal control law is obtained:

$$\boldsymbol{u}_{SDRE} = -\boldsymbol{P}(\boldsymbol{x})^{-1}\boldsymbol{B}(\boldsymbol{x})^T\boldsymbol{K}(\boldsymbol{x})\boldsymbol{x}, \tag{10}$$

where $\boldsymbol{K}(\boldsymbol{x})$ is a state-dependent feedback gain.

4 Numerical experiments

Two rectangular signals with a period of 0.2 s, which are shifted in phase relative by 90°, are created. The input of the nonlinear stepper motor model is the sum of the control signal from the PWM generator and the SDRE compensator.

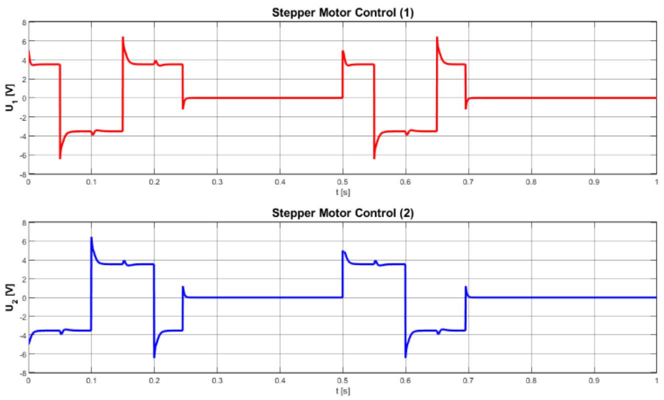

Fig 2: Stepper motor controls

The input matrix $\boldsymbol{B}$ is derived from the physical parameters of the stepper motor. This matrix remain fixed throughout the experiment, whereas $\boldsymbol{A}(\boldsymbol{x})$ matrix is recalculated at each simulation step, based on the current state vector $\boldsymbol{x}(t)$.

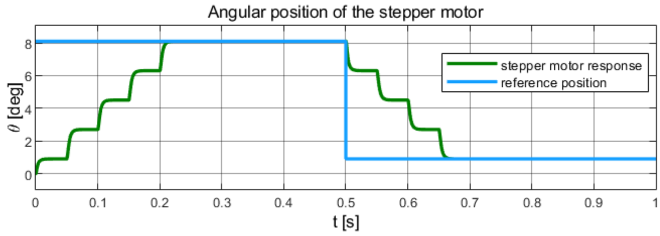

Fig 3: Step response and reference position

The reference value must be a multiple of 1.8°, due to the construction of the stepper motor. The stepper motor cannot achieve a precise 0° position, so its initial offset should be set to either -0.9° or 0.9°, which are the closest achievable positions. The use of SDRE optimal compensation minimizes both energy delivered to and energy lost by the motor.

5 Conclusions

The study focused on addressing the challenges posed by the nonlinear dynamics of stepper motors through a state-dependent control approach. The proposed nonlinear model, with a dynamically updated state matrix, effectively captures the state-dependent behaviour of the stepper motor, which provides a precise position control under complex dynamics. The experiments show, that SDRE control ensures significant reductions in energy delivery and energy loss without compromising motor dynamics. This dual focus on performance and efficiency marks a meaningful advancement in stepper motor control.

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